Effect of temperature dependence of band gap and device constant on $I$–$V$ characteristics of junction diode

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Abstract

A new expression for the $I$–$V$ characteristics of junction diode is proposed considering the temperature dependence of band gap and device constant. Experimental verification of the proposed theory has been done with silicon diode, yellow LED and green LED. It has been shown that theoretically estimated forward voltage across a junction diode can be predicted within 5% with the experimentally obtained forward voltages at different temperatures ($\sim$30°C to 70°C). © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction and theoretical background

The current-voltage in a diode is characterized by a general classical equation and is being used practically for all purpose. The equation contains some temperature dependent terms. However, larger temperature dependence occurs due to reverse saturation current. It is practically impossible to derive an exact expression for the voltage change when the junction temperature changes. The band gap ($E_g$) and device constant ($n$) are generally assumed constants and independent of temperature. It has been reported that device constant also known as ideality factor, varies with temperature for Schottky diode [1] and for LED [2]. Temperature dependence of the energy gap in GaAs and GaP diodes has been reported by Panish and Casey [3] and also by Varshni [4]. It has been shown that the band gap varies with the temperature by a general following relationship [3].

$$E_g = E_g(0) - \alpha T^2 / (T + \beta)$$

where $E_g$ is the energy gap at temperature $T$ and $E_g(0)$ is its value at 0 K and $\alpha$ and $\beta$ are constants. The variation in the energy gap with temperature is believed to arise from two mechanisms. One is a shift in the relative position of the conduction and valence bands due to temperature dependent dilation of the lattice. Theoretical calculation shows that the effect is linear with temperature [4] at high temperatures. At low temperatures, the thermal expansion coefficient is non-linear with $T$. It even becomes negative over a certain temperature interval. Therefore, the dilation effect on the energy gap is also non-linear. The other, which is major contribution, comes from a shift in the relative position of the conduction and valence bands due to temperature dependent electron lattice interaction. Theoretical calculation shows that this leads to a temperature dependence of the following form [4]

$$\Delta E_g \propto T^2 \quad \text{for} \quad T \ll \theta$$

$$\Delta E_g \propto T \quad \text{for} \quad T \gg \theta$$

where $\theta$ is the Debye temperature.

The temperature dependence of the reverse saturation current $I_0$ can be approximated as [5]

$$I_0 = K \exp(-qE_g/nkT)$$

where $K$ is a constant. It is practically difficult to measure the reverse saturation current of some diodes. Its value is even lower of the order of femto-amperes for GaAs or GaP devices whose band gap is large compared to silicon. $I_0$ is estimated from the intercept of ln$I$ versus forward voltage plot. By plotting ln$I_0$ with 1/$T$, $E_g/n$
can be estimated. Experimental measurements have shown that \( \ln I_0 \) varies linearly with \( 1/T \) [2]. If the curve is a straight line, \( E_g/n \) can be obtained from its slope. Since \( E_g \) varies with temperature, \( n \) should also vary in a similar fashion so as to maintain the value of \( E_g/n \) constant. Therefore it can be assumed that

\[ \frac{E_g}{n} = \text{constant} = C \tag{4} \]

Taking this into consideration, it has been reported [2] that \( n \) also varies linearly with temperature and follows a general relationship of the form

\[ n = n(0) - x_1 T^2 / (T + \beta_1) \tag{5} \]

where \( n \) is the device constant at temperature \( T \) and \( n(0) \) is its value at 0 K and \( x_1 \) and \( \beta_1 \) are constants.

Techniques implementing analytical correction of temperature errors over extended temperature range in logarithmic electrometer have been reported [6–8]. The first method is [6] limited to measurement in the current range of \( 10^{-9} \)–\( 10^{-3} \) A while the other method [7] uses LED as a non-linear element for low current applications. A new general expression is reported [9] for the \( I-V \) characteristics of a light emitting diode by which it is possible to determine the forward voltage across diode at actual current and temperature. This expression includes only experimentally obtained and derived parameters than those derived from theory. In this scheme four points are chosen on the measured \( I-V \) characteristics at two temperatures. Fig. 1 shows the plot of \( \log I \) versus forward voltage at two temperatures \( T_1 \) and \( T_2 \). Four points are chosen in the plot as following:

(i) \( V_a \) – output voltage at current \( I_1 \), and temperature \( T_1 \)
(ii) \( V_b \) – output voltage at current \( I_1 \), and temperature \( T_2 \)
(iii) \( V_c \) – output voltage at current \( I_2 \), and temperature \( T_1 \)
(iv) \( V_d \) – output voltage at current \( I_2 \), and temperature \( T_2 \)

The forward voltage \( V_I \) across a junction diode at current \( I \) and at temperature \( T \) can be obtained using the relation suggested based on the experimental values [9].

\[ V_I = V_a + M(T - T_1) + \frac{nkT}{q} \ln \left( \frac{I}{I_1} \right) \left( \frac{T_1}{T} \right)^{3/2} \tag{6} \]

where \( M \) is a constant and is given by

\[ M = \frac{V_a - V_b}{T_1 - T_2} + \frac{3nkT_2}{2q(T_1 - T_2)} \ln \left( \frac{V_c - V_a}{V_d - V_b} \right) \tag{7} \]

and the reverse saturation current \( I_0 \) is given by

\[ I_0 = I_1 \left( \frac{T}{T_1} \right)^{3/2} \exp \left( -\frac{V_a - M(T - T_1)}{(nkT/q)} \right) \tag{8} \]

In deriving the above equations, it has been assumed that \( n \) and \( E_g \) are constants and independent of temperature. In view of the temperature dependence of energy gap \( E_g \) and \( n \), above relations can be modified to account for their temperature dependence. In Fig. 1, considering \( n_1, n_2 \) and \( E_{g1} \) and \( E_{g2} \) as device constants and energy gap values at temperatures \( T_1 \) and \( T_2 \) respectively and we can rewrite

\[ V_I = V_a n \frac{T}{n_1 T_1} - C n \frac{T}{T_1} (T - T_1) + \frac{nkT}{q} \ln \left( \frac{I}{I_1} \right) \left( \frac{T_1}{T} \right)^{3/2} \tag{9} \]

where

\[ C = \frac{E_g}{n} = \frac{E_{g1}}{n_1} = \frac{E_{g2}}{n_2} \]

\[ = \frac{T_1 T_2}{T_1 - T_2} \left[ \frac{V_b}{n_2 T_2} - \frac{V_a}{n_1 T_1} + \frac{k}{q} \ln \left( \frac{T_2}{T_1} \right)^{3/2} \right] \tag{10} \]

and the reverse saturation current \( I_0 \) can be redefined as

\[ I_0 = I_1 \left( \frac{T}{T_1} \right)^{3/2} \exp \left( -\frac{V_a n_1 T}{T_1} + C n \frac{T}{T_1} (T - T_1) \right) \tag{11} \]

The validity of the above equations can be verified by substituting various parameters at four points as mentioned above in text from (i) to (iv) except that \( n = n_1 \), \( n = n_2 \, n = n_1 \) and \( n = n_2 \) to be added in (i)–(iv) conditions respectively. We obtain LHS and RHS in conditions (i) and (iii) and from conditions (ii) and (iv), we obtain

![Image of Fig. 1. I–V characteristics of a junction diode at two temperatures \( T_1 \) and \( T_2 \).](image-url)
\[ V_c = V_a + \frac{n_1 k T_1}{q} \ln \left( \frac{I_2}{I_1} \right) \]  
(13)

and

\[ V_a = V_b + \frac{n_2 k T_2}{q} \ln \left( \frac{I_2}{I_1} \right) \]  
(14)

which confirms with the classical diode equation.

\( I-V \) characteristics of junction diodes of different materials have been measured at different temperatures and it has been shown using proposed equations that the forward voltage at any current and temperature can be predicted within 5\% in the current range \( 10^{-11} \rightarrow 10^{-4} \) A and for the temperature range \(-30^\circ\text{C} \rightarrow 70^\circ\text{C}\).

2. Experimental verification

Different type of junction diodes like silicon diode, yellow LED and green LED were chosen for the experiment to verify the theory. Initially a silicon diode type FJ1 000 (Fairchild) is used for the experiment. It is a pico-ampere low leakage diode. The \( I-V \) characteristics of diode is determined by connecting the diode as a feedback element of the operational amplifier. Low leakage current operational amplifier, OPA104 (Burr Brown) is chosen for the experiment. Standard current source model-261 (Keithley) is used to feed the input current to the log amplifier. \( I-V \) characteristics are obtained at room temperature and also at three other temperatures (Fig. 2). The value of the device constant is obtained from the slope of curve \( \ln I \) versus output voltage of the log amplifier. The reverse saturation current can be obtained from the intercept of the curve. The chosen current values \( I_1 \) and \( I_2 \) are \( 10^{-5} \) and \( 10^{-10} \) A and temperatures \( T_1 \) and \( T_2 \) are \( 30^\circ\text{C} \) and \( 70^\circ\text{C} \) respectively (Fig. 1). Table 1 shows the value of different parameters used in Eq. (9), and the value of \( n \) at different temperatures. It is observed that the value of the device constant \( n \) varies with temperature and is shown in Fig. 3. It is observed that \( n \) varies almost linearly with temperature. The variations of \( I_0 \) with temperature is well known (Eq. (3)). Logarithmic of \( I_0 \) with inverse temperature is plotted in Fig. 4. The variation is found to be linear suggesting \( E_0/n \) to be constant in this temperature

\begin{table}[h]
\centering
\caption{Measured and calculated data for silicon diode (FJ1 000), yellow LED and green LED}
\begin{tabular}{|c|c|c|c|}
\hline
 & Silicon diode & Yellow LED & Green LED \\
\hline
\( I_0 \) (A) & 0.676 & 1.627 & 1.607 \\
\( V_a \) & 0.598 & 1.523 & 1.495 \\
\( V_c \) & 0.358 & 1.01 & 1.008 \\
\( V_a \) & 0.245 & 0.848 & 0.830 \\
\( C \) & 1.11 & 1.06 & 0.968 \\
\hline
\( n \) (\(^\circ\text{C}\)) & & & \\
-30 & 1.1042 & 1.999 & 2.047 \\
-10 & 1.0987 & 1.991 & 2.029 \\
30 & 1.0584 & 1.977 & 1.977 \\
50 & 1.056 & 1.963 & 1.926 \\
70 & 1.049 & 1.937 & 1.900 \\
\hline
\( I_1 \) (A) & \( 10^{-5} \) & \( 10^{-5} \) & \( 10^{-5} \) \\
\( I_2 \) (A) & \( 10^{-10} \) & \( 10^{-10} \) & \( 10^{-10} \) \\
\( T_1 \) (K) & 303 & 303 & 303 \\
\( T_2 \) (K) & 343 & 343 & 343 \\
\hline
\end{tabular}
\end{table}

Fig. 2. \( I-V \) characteristics of a silicon diode (FJ1 000) obtained at different temperatures. The solid line is a result of linear fit.

Fig. 3. Device constant \( n \), as a function of temperature for a silicon diode. The solid line is a result of linear fit.
range of operation. The value of $C (= E_a/n)$ obtained from Eq. (11) for silicon diode is 1.11 which is slightly different from experimentally obtained value 1.05 from $\ln I_n$ versus $1/T$ curve. The forward output voltage as given in Eq. (9) is calculated as a function of current at different temperatures. Fig. 5 shows the % change in forward voltage for silicon diode as a function of current at different temperatures. The % change is defined as

$$\% \text{ change} = \frac{V_{\text{exp}}(T) - V_{\text{theo}}(T)}{V_{\text{exp}}(T)} \times 100$$  \hspace{1cm} (15)$$

where $V_{\text{exp}}(T)$ is experimentally measured voltage at temperature $T$, $V_{\text{theo}}(T)$ is theoretically estimated using Eq. (9) at temperature $T$. The % change is calculated for the earlier model [9] using Eq. (6) at 70°C. It is observed that the proposed Eq. (9) gives better result than given by earlier model using Eq. (6).

Fig. 4. $\log(I_n)$ as a function of inverse temperature for a silicon diode.

Fig. 5. The % change between the experimentally determined output voltage with the theoretically estimated using Eq. (9) at $-10^\circ C$, $50^\circ C$ and $70^\circ C$ and at $70^\circ C$ using Eq. (6). The % change at $70^\circ C$ using Eq. (6) has been denoted as $70^\circ C(O)$ in the figure.

Fig. 6. $I-V$ characteristics of a silicon diode and a LED at two temperatures $T_1$ (303 K) and $T_2$ (343 K).

Fig. 7. The % change between the experimentally determined output voltage with the theoretically estimated using Eq. (9) at $-10^\circ C$, $50^\circ C$ and $70^\circ C$ for a yellow LED.
Experiment is repeated for yellow and green LEDs. Fig. 6 shows the $I$–$V$ characteristics of a yellow LED at two temperatures $T_1$ (303 K) and $T_2$ (343 K). $I$–$V$ characteristics of a silicon diode at these temperatures is also shown for comparison in the same figure. The measured and calculated data used in Eq. (9) for LEDs are also shown in Table 1. The value of $C$ obtained from Eq. (11) for yellow and green LED are 1.06 and 0.968 respectively. The device constant $n$ for both diodes is found to be varying linearly with temperature and decreases with increase of temperature. The % change in forward voltage at different temperatures as a function of current are shown in Figs. 7 and 8 respectively for yellow and green LED. It has been found that the difference is within 5% for both type of LEDs.

In summary, a new expression is proposed for the $I$–$V$ characteristics of junction diode which accounts for the temperature dependence of the band gap and device constant. The expression is a modified version of the earlier expression proposed by Acharya and Vyawahare [9]. Experimental verification of the theory has been made with diodes of different materials. It therefore can be said that this in general will hold good for all junction diodes. It has been shown that forward voltage at any current and temperature can be predicted within 5% in the current range $10^{-11}$–$10^{-4}$ A and for the temperature range $-30^\circ$C to $70^\circ$C.

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References